

Smooth dependence by lag at initial value problems for delay integro-differential equation

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ABSTRACT.

Using the Perov's fixed point theorem and the theorem of fiber generalized contractions obtained by I. A. Rus, here we investigate the smooth dependence by lag for the solution of an initial value problem associated to a nonlinear Volterra integro-differential equation with constant delay. Such equation appears as a model in the spread of infectious diseases with a contact rate that varies seasonally. Sufficient conditions for the smooth dependence by lag of the proportion of infections in population and of the speed of infectivity are derived.

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