

Existence and uniqueness for C_0 -semigroups on the dual of a Banach space

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ABSTRACT.

The main results of this paper are a satisfactory variant of Hille-Yosida theorem and a uniqueness theorem for a C_0 -semigroup on the dual of a Banach space. As application a simplified Hille-Yosida theorem for positive C_0 -semigroups on L^∞ is given. Moreover, the uniqueness of Schrödinger's operator in L^∞ is presented.

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