

Ulam stabilities of ordinary differential equations in a Banach space

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ABSTRACT.

Let $(\mathbb{B}, |\cdot|)$ be a Banach space, $A : \mathbb{B} \rightarrow \mathbb{B}$ be the infinitesimal generator of a C_0 -semigroup, $I := [a, b]$ or $[a, +\infty[$ and $f \in C(I \times \mathbb{B}, \mathbb{B})$. In this paper we present and discuss four types of Ulam stability: Ulam-Hyers stability, generalized Ulam-Hyers stability, Ulam-Hyers-Rassias stability and generalized Ulam-Hyers-Rassias stability for the following differential equation

$$u'(t) = A(u(t)) + f(t, u(t)), \quad t \in I.$$

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