

Approximation theorems in weighted Lorentz spaces

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ABSTRACT.

In this paper we deal with the converse and simultaneous approximation problems of functions possessing derivatives of positive orders by trigonometric polynomials in the weighted Lorentz spaces with weights satisfying the so called Muckenhoupt's A_p condition.

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