On a second order differential equation with piecewise constant mixed arguments

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Abstract.

We prove the existence and uniqueness of solutions of a class of second order differential equations with piecewise constant mixed arguments and we show that the zero solution of Eq. (1.1) is a global attractor. Also, we study some properties of solutions of Eq. (1.1) such as oscillation, nonoscillation, and periodicity.

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