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Stability of Picard iteration for contractive mappings satisfying an implicit relation

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ABSTRACT.

We obtain new and very general stability results for Picard iteration associated to self operators satisfying an implicit relation. Our stability results unify, extend, generalize, enrich and complement a multitude of related stability results from recent literature.

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