

## Sufficient conditions for the existence of some nonoscillatory solutions of third-order nonlinear differential equations

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### ABSTRACT.

The aim of this paper is to study the asymptotic behavior of solutions of nonlinear differential equations of the third-order with quasiderivatives. In particular, we state the sufficient conditions ensuring the existence of some nonoscillatory solutions with a specified asymptotic property as  $t$  tends to infinity. The basic tool used in proving our results is the classical Banach contraction mapping principle.

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