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Sharp form of inequality for the constant e

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ABSTRACT.

(i) We determine the best possible constants α and β such that the inequalities

$$2\left(n+\alpha\right)\left(\frac{2^n n!}{(2n)!}\right)^{1/n} < e \leq 2\left(n+\beta\right)\left(\frac{2^n n!}{(2n)!}\right)^{1/n}$$

are valid for all integers $n \geq 1$.

(ii) Let the sequence v_n be defined by

$$v_n = 2\left(n+\frac{\ln 2}{2}+\frac{a}{n}+\frac{b}{n^2}\right)\left(\frac{2^n n!}{(2n)!}\right)^{1/n}.$$

We determine the values a, b which provide the fastest sequence $(v_n)_{n\geq 1}$ approximating the constant e.

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