Properties of the solutions of those equations for which the Krasnoselskii iteration converges

IOAN A. RUS

Abstract.

Let $(X, +, \mathbb{R}, \rightarrow)$ be a vectorial *L*-space, $Y \subset X$ a nonempty convex subset of *X* and $f : Y \rightarrow Y$ be an operator with $F_f := \{x \in Y \mid f(x) = x\} \neq \emptyset$. Let $0 < \lambda < 1$ and let f_{λ} be the Krasnoselskii operator corresponding to *f*, i.e.,

$$f_{\lambda}(x) := (1 - \lambda)x + \lambda f(x), \ x \in Y.$$

We suppose that f_{λ} is a weakly Picard operator (see I. A. Rus, *Picard operators and applications*, Sc. Math. Japonicae, **58** (2003), No. 1, 191-219). The aim of this paper is to study some properties of the fixed points of the operator f: Gronwall lemmas and comparison lemmas (when $(X, +, \mathbb{R}, \rightarrow, \leq)$) is an ordered *L*-space) and data dependence (when *X* is a Banach space). Some applications are also given.

DEPARTMENT OF MATHEMATICS BABEŞ-BOLYAI UNIVERSITY KOGĂLNICEANU 1, 400084, CLUJ-NAPOCA, ROMANIA *E-mail address*: iarus@math.ubbcluj.ro

2010 Mathematics Subject Classification. 47H10, 47J25, 65J15, 54H25, 37N30, 39A30, 39B12.

Received: 30.10.2011; In revised form: 18.11.2011; Accepted: 23.04.2012

Key words and phrases. Fixed point, weakly Picard operator, Picard operator, ψ -weakly Picard operator, ψ -Picard operator, Krasnoselskii operator, iteration, Gronwall lemma, comparison lemma, data dependence, Ulam-Hyers stability.