

## Explicit integral representations of implicit functions

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### ABSTRACT.

The main goal of the paper is to obtain explicit integral representations of implicit functions whose existence is guaranteed by the implicit function theory. We derive the mentioned representation formulas of inverse functions within a very general framework on general domains in several dimensions.

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