

Oscillatory and periodic solutions in alternately advanced and delayed differential equations

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ABSTRACT.

We examine scalar differential equations with a general piecewise constant argument (DEPCAG). It is shown that the argument deviation generates, under certain conditions, oscillations of the solutions, which is an impossible phenomenon for the corresponding equation without the argument deviations. Criteria for existence of periodic solutions of such equations are discussed.

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