On the existence of normal complement to the Sylow subgroups of some infinite groups

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ABSTRACT.

W. Burnside has proved that if a Sylow p–subgroup P of a finite group G is abelian and $N_G(P) = C_G(P)$, then P has a normal complement, that is G is p–nilpotent. This result has been extended by A. Ballester-Bolinches and R. Esteban-Romero that have shown that if a Sylow p–subgroup P of a finite group G is modular and $N_G(P) = PC_G(P)$, then G is p–nilpotent. In this paper we generalize the latter result to infinite groups. We show that a hyperfinite group G with a Sylow p–subgroup S that is modular and pronormal is p–nilpotent if and only if $N_G(S)$ is p-nilpotent.

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