

## On a heat kernel for the index Whittaker transform

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### ABSTRACT.

We introduce a notion of the heat kernel associated with the index Whittaker transform. We study its differential and mapping properties and a relationship with a family of the corresponding Weierstrass's type transforms.

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