Notes on topological rings

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ABSTRACT.

We prove that every infinite nilpotent ring R admits a ring topology $\mathcal{T}$ for which $(R, \mathcal{T})$ has an open totally bounded countable subring with trivial multiplication. A new example of a compact ring R for which R^2 is not closed, is given. We prove that every compact Bezout domain is a principal ideal domain.

REFERENCES

[1] Arnautov, V., Glavatsky, S. and Mikhalev, A., *Introduction to the theory of topological rings and modules*, Marcel Dekker, Inc, 1996
[2] Choban, M. and Ursul, M., *Applications of the Stone duality in the theory of precompact Boolean rings*, Advances in ring theory, 85-111, Trends Math., Birkhäuser, Springer, Basel AG, Basel, 2010
[3] Dikranjan, D. and Shakhmatov, D., *The Markov-Zariski topology of an abelian group*, J. Algebra, **324** (2010), No. 6, 1125-1158
[4] Engelking, R. and Pełczynski, A., *Remarks on dyadic spaces*, Coll. Math., **11** (1963), 55–63
[5] Hartman, S. and Mycielski, J., *On the imbedding of topological groups into connected topological groups*, Colloq. Math., **5** (1958), 167–169
[6] Hesse, G., *Zur Topologisierbarkeit von Gruppen* (Univ. Hannover, Hannover, 1979)
[7] Koppelberg, S., *Handbook of boolean algebras*, (Monk, J. Donald and R. Bonnet, Eds.), vol. **1**, Elsevier Science Publishers, 1989
[8] Kertesz, A. and Szele, T., *On the existence of non-discrete topologies in infinite abelian groups*, Publ. Math. Debrecen, **3** (1953), 187–189
[9] Lanski, C., *Rings with few nilpotents*, Houston J. Math., **18** (1992), No. 4, 577–590
[10] Malcev, A. I., Topological algebra and Lie groups, In *Mathematics in USSR*, 1917-1947 (A. G. Kurosh, A. I. Markushevici and P. K. Rashevski, Eds.) Moscow-Leningrad, 1948, 134–180
[11] Ol'shanskii, A. Yu., *A note on countable non-topologizable groups*, Vestnik Mosk. Gos. Univ. Mat. Mekh., **103** (1980), No. 3
[12] Shelah, S., On a problem of Kurosh, Jonsson groups and applications, in *Word Problems II* (S. I. Adian, W. W. Boone and G. Higman, Eds.), North-Holland, Amsterdam, 1980, 373–394
[13] Sikorski, R., *Boolean Algebras*, Third edition, Berlin-Göttingen-Heidelberg-New York, 1969
[14] Ursul, M., *Topological Rings Satisfying Compactness Conditions*, Vol. **549**, Kluwer Academic Publishers, Dordrecht-Boston-London, 2002
[15] Witt, E., *Über die kommutatorgruppe kompakter Gruppen*, Rend. Math. e Appl., **14** (1954), 627–630

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