Oscillation of second-order differential equations with a sublinear neutral term

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ABSTRACT.

This paper is concerned with oscillation of a certain class of second-order differential equations with a sublinear neutral term. Two oscillation criteria and two illustrative examples are included. In particular, the results obtained improve those reported in the literature.

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