

Separation problem for the Grushin differential operator in Banach spaces

H. A. ATIA

ABSTRACT.

Our goal in this work is to study the separation problem for the Grushin differential operator formed by

$$Gu = -\frac{1}{2} \left(\frac{\partial^2 u}{\partial x^2} + \frac{x^4}{4} \frac{\partial^2 u}{\partial y^2} \right) + Q(x, y)u(x, y), \quad \forall (x, y) \in \mathbb{R}^2,$$

in the Banach space $H_1(\mathbb{R}^2)$, where the potential $Q(x, y) \in L(\ell_1)$, is a bounded linear operator which transforms at ℓ_1 in value of (x, y) .

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MATHEMATICS DEPARTMENT
RABIGH COLLEGE OF SCIENCE AND ART
KING ABDULAZIZ UNIVERSITY
P. O. BOX 344, RABIGH 21911, SAUDI ARABIA

ZAGAZIG UNIVERSITY
FACULTY OF SCIENCE
MATHEMATICS DEPARTMENT
ZAGAZIG, EGYPT
E-mail address: h.a.atia@hotmail.com