CARPATHIAN J. MATH. 30 (2014), No. 1, 47 - 54
Online version available at http://carpathian.ubm.ro
Print Edition: ISSN 1584 - 2851 Online Edition: ISSN 1843 - 4401

Note on stability of the Cauchy equation – an answer to a problem of Th. M. Rassias

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ABSTRACT.
We give an answer to a problem formulated by Th. M. Rassias in 1991 concerning stability of the Cauchy equation; we also disprove a conjecture of Th. M. Rassias and J. Tabor. In particular, we present a new method for proving stability results for functional equations.

REFERENCES

[1] Aoki, T., *On the stability of the linear transformation in Banach spaces*, J. Math. Soc. Japan, **2** (1950), 64–66
[2] Baker, J. A., *The stability of certain functional equations*, Proc. Amer. Math. Soc., **112** (1991), 729–732
[3] Bourgin, D. G., *Approximately isometric and multiplicative transformations on continuous function rings*, Duke Math. J., **16** (1949), 385–397
[4] Bourgin, D. G., *Classes of transformations and bordering transformations*, Bull. Amer. Math. Soc., **57** (1951), 223–237
[5] Brzdęk, J., A note on stability of additive mappings, in *Stability of Mappings of Hyers-Ulam Type* (Th. M. Rassias, J. Tabor, Eds.), Hadronic Press, Inc., Florida, 1994, 19–22
[6] Brzdęk, J., *On a method of proving the Hyers-Ulam stability of functional equations on restricted domains*, Austral. J. Math. Anal. Appl., **6** (2009), No. 1, Art. 4, 10 pp
[7] Brzdęk, J., Chudziak, J. and Páles, Z., *A fixed point approach to stability of functional equations*, Nonlinear Anal., **74** (2011), 6728–6732
[8] Brzdęk, J. and Ciepliński, K., *A fixed point approach to the stability of functional equations in non-Archimedean metric spaces*, Nonlinear Anal., **74** (2011), 6861–6867
[9] Cădariu, L., Găvruţa, L. and Găvruţa, P., *Fixed points and generalized Hyers-Ulam stability*, Abstr. Appl. Anal., 2012 (2012), Art. ID 712743, 10 pp
[10] Cădariu, L. and Radu, V., *Fixed point methods for the generalized stability of functional equations in a single variable*, Fixed Point Theory Appl., 2008 (2008), Art. ID 749392, 15 pp
[11] Dhombres, J. and Ger, R., *Équations de Cauchy conditionnelles*, C. R. Acad. Sc. Paris Sér. A, **280** (1975), 513–515
[12] Dhombres, J. and Ger, R., *Conditional Cauchy equations*, Glasnik Mat. III, **13** (1978), No. 33, 39–62
[13] Forti, G.-L., *Comments on the core of the direct method for proving Hyers-Ulam stability of functional equations*, J. Math. Anal. Appl., **295** (2004), 127–133
[14] Forti, G.-L., *Elementary remarks on Ulam-Hyers stability of linear functional equations*, J. Math. Anal. Appl., **328** (2007), 109–118
[15] Forti, G.-L. and Sikorska, J., *Variations on the Drygas equation and its stability*, Nonlinear Anal., **74** (2011), 343–350
[16] Gajda, Z., *On stability of additive mappings*, Int. J. Math. Math. Sci., **14** (1991), 431–434
[17] Găvruţa, P., *A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings*, J. Math. Anal. Appl., **184** (1994), 431–436
[18] Găvruţa, P., *On the approximately linear mappings*, Bull. Appl. Math. (Budapest), **80** (1996), 317–322
[19] Găvruţa, P., *On the Hyers-Ulam-Rassias stability of the quadratic mappings*, Nonlinear Funct. Anal. Appl., **9** (2004), 415–428
[20] Găvruţa, P., On the Hyers-Ulam-Rassias stability of mappings, in *Recent Progress in Inequalities*, (G. V. Milovanovic, ed.), Kluwer Academic Publishers, 1998, 465–469

Received: 15.05.2012; In revised form: 05.01.2013; Accepted: 21.06.2013
2010 *Mathematics Subject Classification.* 39B62, 39B82.
Key words and phrases. *Hyers-Ulam-Rasssias stability, Cauchy equation, additive function.*

[21] Ger, R., On functional inequalities stemming from stability equations, in *General Inequalities 6*, (W. Walter, Ed.), 227-240, ISNM 103, Birkhäuser, Basel, 1992

[22] Hyers, D. H., *On the stability of the linear functional equation*, Proc. Natl. Acad. Sci. U.S.A., **27** (1941), 222–224

[23] Hyers, D. H., *Transformations with bounded mth differences*, Pacific J. Math., **11** (1961), 591–602

[24] Hyers, D. H., Isac, G. and Rassias, Th. M., *Stability of Functional Equations in Several Variables*, Birkhäuser, Boston, 1998

[25] Hyers, D. H., Isac, G. and Rassias, Th. M., *On the asymptoticity aspect of Hyers-Ulam stability of mappings*, Proc. Amer. Math. Soc., **126** (1998), 425–430

[26] Hyers, D. H. and Rassias, Th. M., *Approximate homomorphisms*, Aequationes Math., **44** (1992), 125–153

[27] Hyers, D. H. and Ulam, S. M., *On approximate isometries*, Bull. Amer. Math. Soc., **51** (1945), 288–292

[28] Hyers, D. H. and Ulam, S. M., *Approximate isometries of the space of continuous functions*, Ann. of Math., **48** (1947), No. 2, 285–289

[29] Hyers, D. H. and Ulam, S. M., *Approximately convex functions*, Proc. Amer. Math. Soc., **3** (1952), 821–828

[30] Isac, G. and Rassias, Th. M., *On the Hyers-Ulam stability of ψ-additive mappings*, J. Approx. Theory, **72** (1993), 131–137

[31] Isac, G. and Rassias, Th. M., *Stability of ψ-additive mappings: Applications to nonlinear analysis*, Int. J. Math. Math. Sci., **19** (1996), 219–228

[32] Jabłoński, W., *Stability of homogeneity almost everywhere*, Acta Math. Hungar., **117** (2007), 219–229

[33] Jung, S.-M., *Hyers-Ulam-Rassias stability of functional equations*, Dynam. Systems Appl., **6** (1997), 541–566

[34] Jung, S.-M., *A fixed point approach to the stability of isometries*, J. Math. Anal. Appl., **329** (2007), 879–890

[35] Jung, S.-M., *Hyers-Ulam-Rassias Stability of Functional Equations in Nonlinear Analysis*, Springer Optimization and Its Applications, vol. **48**, Springer, New York-Dordrecht-Heidelberg-London, 2011

[36] Jung, S.-M. and Kim, T.-S., *A fixed point approach to the stability of the cubic functional equation*, Bol. Soc. Mat. Mexicana, **12** (2006), No. 3, 51–57

[37] Jung, S.-M., Kim, T.-S. and Lee, K.-S., *A fixed point approach to the stability of quadratic functional equation*, Bull. Korean Math. Soc., **43** (2006), 531–541

[38] Jung, Y.-S. and Chang, I.-S., *The stability of a cubic functional equation and fixed point alternative*, J. Math. Anal. Appl., **306** (2005), 752–760

[39] Kominek, Z., *On a local stability of the Jensen functional equation*, Demonstratio Math., **22** (1989), 499–507

[40] Kuczma, M., *An Introduction to the Theory of Functional Equations and Inequalities, Cauchys Equation and Jensens Inequality*, Birkhuser, 2nd edition, 2009

[41] Losonczi, L., *On the stability of Hosszú's functional equation*, Results Math., **29** (1996), 305–310

[42] Mirzavaziri, M. and Moslehian, M. S., *A fixed point approach to stability of a quadratic equation*, Bull. Braz. Math. Soc., New Series, **37** (2006), 361–376

[43] Moszner, Z., *On the stability of functional equations*, Aequationes Math., **77** (2009), 33–88

[44] Pólya, Gy. and Szegö, G., *Aufgaben und Lehrsätze aus der Analysis I*, Verlag von Julius Springer, Berlin, 1925

[45] Radu, V., *The fixed point alternative and the stability of functional equations*, Fixed Point Theory, **4** (2003), 91–96

[46] Rassias, Th. M., *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc., **72** (1978), 297–300

[47] Rassias, Th. M., *Problem*, Aequationes Math., **39** (1990), 309

[48] Rassias, Th. M., *On a modified Hyers-Ulam sequence*, J. Math. Anal. Appl., **158** (1991), 106–113

[49] Rassias, Th. M., *On the stability of the quadratic functional equation and its applications*, Studia Univ. Babeş-Bolyai Math., **43** (1998), No. 3, 89–124

[50] Rassias, Th. M., *The problem of S.M. Ulam for approximately multiplicative mappings*, J. Math. Anal. Appl., **246** (2000), 352–378

[51] Rassias, Th. M., *On the stability of functional equations and a problem of Ulam*, Acta Appl. Math., **62** (2000), 23–130

[52] Rassias, Th. M., *On the stability of functional equations in Banach spaces*, J. Math. Anal. Appl., **251** (2000), 264–284

[53] Rassias, Th. M. and Šemrl, P., *On the behavior of mappings which do not satisfy Hyers-Ulam stability*, Proc. Amer. Math. Soc., **114** (1992), 989–993

[54] Rassias, Th. M. and Šemrl, P., *On the Hyers-Ulam stability of linear mappings*, J. Math. Anal. Appl., **173** (1993), 325–338

[55] Rassias, Th. M. and Tabor, J., *What is left of Hyers-Ulam stability?*, J. Nat. Geometry, **1** (1992), 65–69

[56] Rassias, Th. M. and Tabor, J., On approximately additive mappngs in Banach spaces, in *Stability of Mappings of Hyers-Ulam Type* (Th. M. Rassias, J. Tabor, Eds.), Hadronic Press, Palm Harbor, 1994, 127–134

[57] Rätz, J., On approximately additive mappings, in *General Inequalities 2* (E. F. Beckenbach, Ed.), Birkhäuser, Basel, 1980, 233–251

[58] Šemrl, P., The stability of approximately additive mappings, in *Stability of Mappings of Hyers-Ulam Type*, (Th. M. Rassias, J. Tabor, Eds.), Hadronic Press, Palm Harbor, 1994, 135–140

[59] Sikorska, J., *On a direct method for proving the Hyers-Ulam stability of functional equations*, J. Math. Anal. Appl., **372** (2010), 99–109

[60] Skof, F., On the stability of functional equations on a restricted domain and related topics, in *Stability of Mappings of Hyers-Ulam Type*, (Th. M. Rassias, J. Tabor, Eds.), Hadronic Press, Palm Harbor, 1994, 141–151

[61] Ulam, S. M., *Problems in Modern Mathematics*, Science Editions, John-Wiley & Sons Inc., New York, 1964

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