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On a generalization of the Levin-May Theorem

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ABSTRACT.

The paper discusses a distribution of the zeros of the polynomial

$$p(\lambda) \equiv \lambda^{k+1} - \lambda^k + q, \qquad q \in \mathbb{R}, \quad k \in \mathbb{Z}^+$$

with respect to the unit circle. This problem is of theoretic as well as practical importance which motivated S. A. Levin and R. May to formulate a necessary and sufficient condition guaranteeing the location of all the zeros of $p(\lambda)$ inside the unit circle. We give a simple alternate proof of their criterion and, as the main result, present a complete list of all possible zero distributions of $p(\lambda)$ with respect to this circle.

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REFERENCES

[1] Bellen, A. and Zennaro, M., *Numerical Methods For Delay Differential Equations*, Numerical Mathematics and Scientific Computation, The Clarendon Press, Oxford University Press, New York, 2003
[2] Bistritz, Y., *Zero location of polynomials with respect to the unit-circle unhampered by nonessential singularities*, IEEE Trans. Circuits Syst., **49** (2002), No. 3, 305–314
[3] Čermák, J., Jánský, J. and Kundrát, P., *On necessary and sufficient conditions for the asymptotic stability of higher order linear difference equations*, J. Difference Equ. Appl., **18** (2012), No. 11, 1781–1800
[4] Elaydi, S., *An Introduction to Difference Equations*, 3rd ed., Undergraduate Texts in Mathematics, Springer, New York, 2005
[5] Jury, E. I., *Theory and Application of The $\mathcal{Z}$-Transform Methods*, Wiley, New York, 1964
[6] Kim, S. H. and Park, C. W., *On the zeros of certain self-reciprocal polynomials*, J. Math. Anal. Appl., **339** (2008), 240–247.
[7] Kipnis, M. M. and Nigmatullin, R. M., *Stability of the trinomial linear difference equations with two delays*, Autom. Remote Control, **65** (2004), No. 11, 1710–1723
[8] Kipnis, M. M. and Levitskaya, I. S., Stability of delay difference and differential equations: Similarities and distinctions, in *Proceedings of the International Conference Difference Equations, Special Functions and Orthogonal Polynomials*, Munich, Germany, 25–30 July 2005, World Scientific, Singapore, 2007, 315–324
[9] Kuruklis, S. A., *The asymptotic stability of $x_{n+1} - ax_n + bx_{n-k} = 0$*, J. Math. Anal. Appl., **188** (1994), 719–731
[10] Levin, S. A. and May, R., *A note on difference–delay equations*, Theor. Popul. Biol., **9** (1976), 178–187
[11] Marden, M., *Geometry of Polynomials*, Mathematical Surveys and Monographs, No. 3, Providence, 1966
[12] Matsunaga, H. and Hajiri, C., *Exact stability sets for a linear difference system with diagonal delay*, J. Math. Anal. Appl., **369** (2010), 616–622
[13] Pal, D. and Kailath, T., A generalization of the Schur-Cohn test: the singular cases, in *Proceedings of the 33rd Midwest Symposium on Circuits and Systems*, Calgary, Canada, 12-15 August, 1990, 64–67

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