Fixed points of set-valued contractions in partial metric spaces endowed with a graph

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ABSTRACT.

Hassen, Abbas and Vetro [H. Aydi, M. Abbas and C. Vetro, *Partial Hausdorff metric and Nadler's fixed point theorem on partial metric spaces*, Topology and its App., **159** (2012), 3234–3242] introduced the concept of a partial Hausdorff-Pompeiu metric and proved Nadler's theorem in this context. Employing the notion of a partial Hausdorff-Pompeiu metric, we investigate the existence of fixed points of set-valued mappings on partial metric spaces endowed with a graph. Our results extend some recent theorems in the literature.

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