

Asymptotic expansions for the ratio of gamma functions

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ABSTRACT.

The main object of this paper is to establish a class of the asymptotic expansions for the ratio of gamma functions $\frac{\Gamma(x+1)}{\Gamma(x+\frac{1}{2})}$, by mainly using Bell polynomials and the partition function. This unifies several approximation formulas for the ratio of gamma functions due to Mortici.

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Received: 03.01.2013; In revised form: 05.09.2013; Accepted: 15. 01.2014
2010 Mathematics Subject Classification. 33B15, 41A60.

Key words and phrases. Gamma function, wallis ratio, asymptotic formula, Bernoulli numbers, Bell polynomials, partition function.

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