

## Some results of differentiability for the solution of an integral equations system

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### ABSTRACT.

Using the fixed point theorem given by [Rus, I. A., *A Fiber generalized contraction theorem and applications*, *Mathematica*, **41(64)** (1999), No. 1, 85–90] and an idea of [Sotomayor, J., *Smooth dependence of solution of differential equation on initial data: a simple proof*, *Bol. Soc. Brasil.*, **4** (1973), No. 1, 55–59] we establish some conditions of differentiability of the solution for the following system of integral equations:

$$x(t) = \int_a^b K(t, s) \cdot h(s, x(s), x(a), x(b)) ds + f(t), \quad t \in [a, b],$$

and such we obtain two theorems of differentiability. Finally, two examples are given.

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