

## Fixed points for $\alpha$ - $\psi$ -Suzuki contractions with applications to integral equations

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### ABSTRACT.

Recently, Suzuki [Proc. Amer. Math. Soc. **136** (2008), 1861–1869] proved a fixed point theorem that is a generalization of the Banach contraction principle and characterized the metric completeness. Paesano and Vetro [Topology Appl., **159** (2012), 911–920] proved an analogous fixed point result on a partial metric space. In this paper we prove some fixed point results for Suzuki- $\alpha$ - $\psi$ -contractions and Suzuki- $\varphi_\theta$ - $\psi_r$ -contractions on a complete partially ordered metric space. Moreover, some examples and an application to integral equations are provided to illustrate the usability of the obtained results.

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