

On the L^∞ -uniqueness of multidimensional Nelson's diffusion

LUDOVIC DAN LEMLE

ABSTRACT.

In this paper we study the uniqueness in the sense of the essential self-adjointness for the generator of Nelson's diffusion on L^∞ . As consequence it is obtained the L^1 -uniqueness of weak solutions for the associated Fokker-Planck-Kolmogorov equation.

1. FRAMEWORK

Acknowledgments. I am grateful to the anonymous referees for their careful reading and for useful suggestions. This work was partially supported by the strategic grant POSDRU/21/1.5/G/13798, inside POSDRU Romania 2007-2013, co-financed by the European Social Fund - Investing in People.

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Received: 04.02.2013; In revised form: 03.10.2013; Accepted: 15.01.2014
2010 *Mathematics Subject Classification.* 60J60, 81Q10, 35Q84, 35Q30.

Key words and phrases. L^∞ -uniqueness, diffusion, Fokker-Planck equation, weak solution.

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DEPARTMENT OF ELECTRICAL ENGINEERING AND INDUSTRIAL INFORMATICS
"POLITEHNICA" UNIVERSITY OF TIMIȘOARA
REVOLUȚIEI 5, 331128 HUNEDOARA, ROMANIA
E-mail address: dan.lemle@fih.upt.ro