

Operators commuting with multi-parameter shift semigroups

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ABSTRACT.

Using operators of cross-correlation with ultradistributions supported by a positive cone, we describe a commutative algebra of shift-invariant continuous linear operators, commuting with contraction multi-parameter semigroups over a Banach space. Thereby, we generalize classic Schwartz's and Hörmander's theorems on shift-invariant operators.

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