On the Maksa-Volkmann functional inequality $|f(x+y)| \geq |f(x) + f(y)|$ when the range of f is a space of functions

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ABSTRACT.

P. Volkmann functional inequality $|f(x+y)| \geq |f(x) + f(y)|$ is extended to functions $f : G \to \mathfrak{F}(X, E)$ where G is an additive group and $\mathfrak{F}(X, E)$ is the space of functions from a set X to a linear normed space E. As a corollary one proves that an operator $T : C(X, K) \to C(X, K)$ which satisfies the functional inequality $|T(f+g)| \geq |T(f) + T(g)|$, $f, g \in C(X, K)$ is additive. Here we denoted by X a compact topological space, K is $\mathbb{R}$ or $\mathbb{C}$ and $C(X, K)$ is the linear space of continuous functions defined on X with values in K.

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