

Solutions of variational inequality problems in the set of fixed points of pseudocontractive mappings

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ABSTRACT.

We introduce an iterative process which converges strongly to a solution of the variational inequality problems for η -inverse strongly accretive mappings in the set of fixed points of pseudocontractive mappings. Our theorems improve and unify most of the results that have been proved for this important class of nonlinear operators.

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