Oscillation of even-order neutral differential equations via comparison principles

J. Džurina and B. Baculíková

Abstract.

In the paper we offer oscillation criteria for even-order neutral differential equations

$$\left(r(t)\left[z^{(n-1)}(t)\right]^{\gamma} \right)' + q(t)x^{\gamma}(\sigma(t)) = 0,$$

where $z(t) = x(t) + p(t)x(\tau(t))$. Establishing a generalization of Philos and Staikos lemma, we introduce new comparison principles for reducing the examination of the properties of the higher order differential equation onto oscillation of the first order delay differential equations. The results obtained are easily verifiable.

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Department of Mathematics
Technical University of Košice
Letná 9, 042 00 Košice, Slovakia
E-mail address: blanka.baculikova@tuke.sk
E-mail address: jozef.dzurina@tuke.sk