

On the crossing numbers of Cartesian products of paths with special graphs

MARIÁN KLEŠČ, DANIELA KRAVECOVÁ and JANA PETRILLOVÁ

ABSTRACT.

There are known exact results of the crossing numbers of the Cartesian product of all graphs of order at most four with paths, cycles and stars. Moreover, for the path P_n of length n , the crossing numbers of Cartesian products $G \square P_n$ for all connected graphs G on five vertices and for forty graphs G on six vertices are known. In this paper, we extend these results by determining the crossing numbers of the Cartesian products $G \square P_n$ for six other graphs G of order six.

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Corresponding author: Daniela Kravecová; daniela.kravecova@tuke.sk

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TECHNICAL UNIVERSITY OF KOŠICE

FACULTY OF ELECTRICAL ENGINEERING AND INFORMATICS

DEPARTMENT OF MATHEMATICS AND THEORETICAL INFORMATICS

NĚMCOVEJ 32, 042 00, KOŠICE, SLOVAKIA

E-mail address: marian.klesc@tuke.sk

E-mail address: daniela.kravecova@tuke.sk

E-mail address: jana.petrillova@tuke.sk