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Greedoids on vertex sets of B-joins of graphs

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ABSTRACT.

Let $\Psi(G)$ be the family of all local maximum stable sets of graph G, i.e., $S \in \Psi(G)$ if S is a maximum stable set of the subgraph induced by $S \cup N(S)$, where $N(S)$ is the neighborhood of S. It was shown that $\Psi(G)$ is a greedoid for every forest G [15]. The cases of bipartite graphs, triangle-free graphs, and well-covered graphs, were analyzed in [16, 17, 18, 19, 20, 24].

If G_1, G_2 are two disjoint graphs, and B is a bipartite graph having $E(B)$ as an edge set and bipartition $\{V(G_1), V(G_2)\}$, then by B-join of G_1, G_2 we mean the graph $B(G_1, G_2)$ whose vertex set is $V(G_1) \cup V(G_2)$ and edge set is $E(G_1) \cup E(G_2) \cup E(B)$.

In this paper we present several necessary and sufficient conditions for $\Psi(B(G_1, G_2))$ to form a greedoid, an antimatroid, and a matroid, in terms of $\Psi(G_1)$, $\Psi(G_2)$ and $E(B)$.

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