CARPATHIAN J. MATH. Online version available at http://carpathian.ubm.ro
31 (2015), No. 1, 1 - 10 Print Edition: ISSN 1584 - 2851 Online Edition: ISSN 1843 - 4401

On Montel's theorem in several variables

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ABSTRACT.

Recently, the first author of this paper, used the structure of finite dimensional translation invariant subspaces of $C(\mathbb{R},\mathbb{C})$ to give a new proof of classical Montel's theorem, about continuous solutions of Fréchet's functional equation $\Delta_h^m f = 0$, for real functions (and complex functions) of one real variable. In this paper we use similar ideas to prove a Montel's type theorem for the case of complex valued functions defined over the discrete group $\mathbb{Z}^d$. Furthermore, we also state and demonstrate an improved version of Montel's Theorem for complex functions of several real variables and complex functions of several complex variables.

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Received: 30.10.2013; In revised form: 24.03.2014; Accepted: 23.04.2014
2010 *Mathematics Subject Classification.* 47A15, 46F05, 46F10, 39B22, 39B32.
Key words and phrases. *Functional equations, Montel's theorem, invariant subspaces.*
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