

Expanding the applicability of secant-like methods for solving nonlinear equations

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ABSTRACT.

We use the method of recurrent functions to provide a new semilocal convergence analysis for secant-like methods in order to approximate a locally unique solution of a nonlinear equation in a Banach space setting. Our sufficient convergence criteria are weaker than in earlier studies such as [18, 19, 20, 21, 25, 26]. Therefore, the new approach has a larger convergence domain and uses the same constants. A numerical example involving a nonlinear integral equation of mixed Hammerstein type is given to illustrate the advantages of the new approach. Another example of nonlinear integral equations is presented to show that the old convergence criteria are not satisfied but the new convergence are satisfied.

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