

## On some invariant geometric properties in Hele-Shaw flows with small surface tension

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### ABSTRACT.

In this paper, by applying methods from complex analysis, we analyse the time evolution of the free boundary of a viscous fluid for planar flows in Hele-Shaw cells under injection in the non-zero surface tension case. We study the invariance in time of  $\alpha$ -convexity (for  $\alpha \in [0, 1]$  this is a geometric property which provides a continuous passage from starlikeness to convexity) for bounded domains. In this case we show that the  $\alpha$ -convexity property of the moving boundary in a Hele-Shaw flow problem with small surface tension is preserved in time for  $\alpha \leq 0$ . For unbounded domains (with bounded complement) we prove the invariance in time of convexity.

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