

Homogenization of a class of elliptic problems with nonlinear boundary conditions in domains with small holes

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ABSTRACT.

We consider a class of second order elliptic problems in a domain of \mathbb{R}^N , $N > 2$, ε -periodically perforated by holes of size $r(\varepsilon)$, with $r(\varepsilon)/\varepsilon \rightarrow 0$ as $\varepsilon \rightarrow 0$. A nonlinear Robin-type condition is prescribed on the boundary of some holes while on the boundary of the others as well as on the external boundary of the domain, a Dirichlet condition is imposed. We are interested in the asymptotic behavior of the solutions as $\varepsilon \rightarrow 0$. We use the periodic unfolding method introduced in [Cioranescu, D., Damllamian, A. and Griso, G., *Periodic unfolding and homogenization*, C. R. Acad. Sci. Paris, Ser. I, **335** (2002), 99–104]. The method allows us to consider second order operators with highly oscillating coefficients and so, to generalize the results of [Cioranescu, D., Donato, P. and Zaki, R., *Asymptotic behavior of elliptic problems in perforated domains with nonlinear boundary conditions*, *Asymptot. Anal.*, Vol. **53** (2007), No. 4, 209–235].

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