

On spline collocation and the Hilbert transform

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ABSTRACT.

In this paper we examine a relationship between the spline collocation projection operator π_n and the Hilbert singular integral operator \mathcal{H}_0 . We use Fourier analysis to prove that under certain conditions, a commutator property holds between the two operators. More specifically, we show that for $u \in H^t$, $\|(\pi_n \mathcal{H}_0 - \mathcal{H}_0 \pi_n)u\|_t \leq Ch^\lambda \|u\|_s$ (where $h = 1/n$), for some t, s and $\lambda \in \mathbb{R}$.

REFERENCES

- [1] Adams, R. A., *Sobolev Spaces*, Academic Press, New York/San Francisco/London, 1975
- [2] Anttila, J., *A spline collocation method for parabolic pseudodifferential equations*, J. Comput. Appl. Math., **140** (2002), No. 1-2, 41–61
- [3] Arnold, D. N. and Wendland, W. L., *The Convergence of Spline Collocation for Strongly Elliptic Equations on Curves*, Numer. Math., **47** (1985), No. 3, 317–341
- [4] Efendiev, M. A. and Wendland, W. L., *Nonlinear Riemann-Hilbert problems with Lipschitz continuous boundary data: Doubly connected domains*, R. Soc. Lond. Proc. Ser. A Math. Phys. Eng. Sci., **459** (2003), 945–955
- [5] Grigorieff, R. D., *Additional order convergence in qalocation for elliptic boundary integral equations*, J. Integral Equations Appl., **23** (2011), No. 3, 383–419
- [6] King, F. W., *Hilbert Transforms: Volume 1*, Cambridge University Press, 2009
- [7] Micula, Gh. and Micula, S., *Handbook of Splines*, Kluwer Academic Publishers, Dordrecht/Boston/London, 1999
- [8] Micula, S., *On Superconvergent Spline Collocation Methods for the Radiosity Equation*, Stud. Univ. Babeş-Bolyai Math., **51** (2006), No. 4, 145–156
- [9] Micula, S. and Micula, Gh., *On the Superconvergent Spline Collocation Methods for the Fredholm Integral Equations on Surfaces*, Math. Balkanica (N.S.), **19** (2005), No. 1-2, 155–166
- [10] Prößdorf, S. and Schneider, R., *Spline approximation methods for multidimensional periodic pseudodifferential equations*, Integral Equations and Operator Theory, **15** (1992), No. 4, 626–672
- [11] Saranen, J. and Vainikko, G., *Periodic Integral and Pseudodifferential Equations with Numerical Approximation*, Springer-Verlag, Berlin/Heidelberg/New York, 2002
- [12] Saranen, J. and Wendland, W. L., *The Fourier series representation of pseudodifferential operators on closed curves*, Complex Variables Theory Appl., **8** (1987), No. 1-2, 55–64

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Received: 03.07.2013; In revised form: 08.01.2014; Accepted: 23.04.2014
2010 *Mathematics Subject Classification*. 35J60, 41A15, 42A16, 46F12, 47G30, 65R10.
Key words and phrases. *Spline collocation, Hilbert transform, Fourier analysis.*