

# Qualitative results on the convergence of the $q$ -Bernstein polynomials

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## ABSTRACT.

Despite many common features, the convergence properties of the Bernstein and the  $q$ -Bernstein polynomials are not alike. What is more, the cases  $0 < q < 1$  and  $q > 1$  are not similar to each other in terms of convergence. In this work, new results demonstrating the striking differences which may occur in those convergence properties are presented.

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