

A note on fixed point theorems in metric spaces

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ABSTRACT.

In this paper, we show that the existence of fixed points in some known fixed point theorems in the literature is a consequence of the Banach contraction principle.

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