Bounds of the second degree cumulative frontier gaps of functions with generalized convexity

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ABSTRACT.

We prove that the set of second degree cumulative frontier gaps, via fractional integrals of positive orders, of twice differentiable functions having generalized convexity at the level of the second derivative is upper bounded. A sharp Hermite-Hadamard type inequality via fractional integrals leads to an evaluation of this upper bound, which does not depend on the order of the fractional integration. Six particular generalized convexity properties are investigated from the point of view of this boundary property.

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