

## Extensions of Perov theorem

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### ABSTRACT.

[Perov, A. I., *On Cauchy problem for a system of ordinary differential equations*, (in Russian), Priblizhen. Metody Reshen. Difer. Uravn., **2** (1964), 115-134] used the concept of vector valued metric space and obtained a Banach type fixed point theorem on such a complete generalized metric space. In this article we study fixed point results for the new extensions of Banach's contraction principle to cone metric space, and we give some generalized versions of the fixed point theorem of Perov. As corollaries some results of [Zima, M., *A certain fixed point theorem and its applications to integral-functional equations*, Bull. Austral. Math. Soc., **46** (1992), 179-186] and [Borkowski, M., Bugajewski, D. and Zima, M., *On some fixed-point theorems for generalized contractions and their perturbations*, J. Math. Anal. Appl., **367** (2010), 464-475] are generalized for a Banach cone space with a non-normal cone. The theory is illustrated with some examples.

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