

Some examples of division symbol algebras of degree 3 and 5

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ABSTRACT.

In this paper we provide an algorithm to compute the product between two elements in a symbol algebra of degree n and we find an octonion algebra (in general, without division) in a symbol algebra of degree three. Moreover, using MAGMA software, we will provide some examples of division symbol algebras of degree 3 and of degree 5.

Acknowledgements. Since, the second author had the first ideas for this paper in 2012 during her visit at Centre de Recerca Matemàtica, Barcelona (Spain), as a short term visiting researcher, she thanks the staff of Centre de Recerca Matemàtica for their hospitality and support. This paper is supported by the grant of CNCS (Romanian National Council of Research): PN-II-ID-WE-2012-4-169, for the first author. We also want thank to referees for their suggestions which helped us to improve this paper.

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Received: 20.11.2013; In revised form: 26.05.2014; Accepted: 31.05.2014

2010 *Mathematics Subject Classification.* 16H05, 17A01, 17A35, 15A24, 15A06, 16G30, 1R52, 11R37, 11B39.

Key words and phrases. *Symbol algebras, quaternion algebras, octonion algebras, cyclotomic fields, Kummer fields, ideals class group.*

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