

The metric dimension of strong product graphs

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ABSTRACT.

For an ordered subset $S = \{s_1, s_2, \dots, s_k\}$ of vertices in a connected graph G , the metric representation of a vertex u with respect to the set S is the k -vector $r(u|S) = (d_G(u, s_1), d_G(u, s_2), \dots, d_G(u, s_k))$, where $d_G(x, y)$ represents the distance between the vertices x and y . The set S is a metric generator for G if every two different vertices of G have distinct metric representations with respect to S . A minimum metric generator is called a metric basis for G and its cardinality, $dim(G)$, the metric dimension of G . It is well known that the problem of finding the metric dimension of a graph is NP-Hard. In this paper we obtain closed formulae and tight bounds for the metric dimension of strong product graphs.

REFERENCES

- [1] Brigham, R. C., Chartrand, G., Dutton, R. D. and Zhang, P., *Resolving domination in graphs*, Math. Bohem., **128** (2003), 25–36
- [2] Cáceres, J., Hernando, C., Mora, M., Pelayo, I. M., Puertas, M. L., Seara, C. and Wood, D. R., *On the metric dimension of Cartesian product of graphs*, SIAM J. Discrete Math., **21** (2007), 273–302
- [3] Chartrand, G., Eroh, L., Johnson, M. A. and Oellermann, O. R., *Resolvability in graphs and the metric dimension of a graph*, Discrete Appl. Math., **105** (2000), 99–113
- [4] Chartrand, G., Salehi, E. and Zhang, P., *The partition dimension of a graph*, Aequationes Math., **59** (2000), 45–54
- [5] Harary, F. and Melter, R. A., *On the metric dimension of a graph*, Ars Combin., **2** (1976), 191–195
- [6] Imrich, W. and Klavžar, S., *Product Graphs, Structure and Recognition*, Wiley-Interscience, 2000
- [7] Johnson, M. A., *Structure-activity maps for visualizing the graph variables arising in drug design*, J. Biopharm. Statist., **3** (1993), 203–236
- [8] Johnson, M. A., *Browsable structure-activity datasets*, in Advances in Molecular Similarity, (R. Carbó-Dorca and P. Mezey, Eds.), JAI Press Connecticut, (1998) pp. 153–170
- [9] Khuller, S., Raghavachari, B. and Rosenfeld, A., *Landmarks in graphs*, Discrete Appl. Math., **70** (1996), 217–229
- [10] Kratica, J., Kovačević-Vujčić, V., Čangalović, M. and Stojanović, M., *Minimal doubly metric generators and the strong metric dimension of Hamming graphs*, Appl. Anal. Discrete Math., **6** (2012), 63–71
- [11] Kuziak, D., Yero, I. G. and Rodríguez-Velázquez, J. A., *On the strong metric dimension of corona product graphs and join graphs*, Discrete Appl. Math., **161** (2013), 1022–1027
- [12] May, T. R. and Oellermann, O. R., *The strong dimension of distance-hereditary graphs*, J. Combin. Math. Combin. Comput., **76** (2011), 59–73
- [13] Oellermann, O. R. and Peters-Fransen, J., *The strong metric dimension of graphs and digraphs*, Discrete Appl. Math., **155** (2007), 356–364
- [14] Okamoto, F., Phinezyn, B. and Zhang, P., *The local metric dimension of a graph*, Math. Bohem., **135** (2010), 239–255
- [15] Sebő, A. and Tannier, E., *On metric generators of graphs*, Math. Oper. Res., **29** (2004), 383–393
- [16] Slater, P. J., *Leaves of trees*, Proceeding of the 6th Southeastern Conference on Combinatorics, Graph Theory, and Computing, Congr. Numer., **14** (1975), 549–559

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- [17] Yero, I. G. and Rodríguez-Velázquez, J. A., *A note on the partition dimension of Cartesian product graphs*, Appl. Math. Comput., **217** (2010), 3571–3574
- [18] Yero, I. G., Kuziak, D. and Rodríguez-Velázquez, J. A., *On the metric dimension of corona product graphs*, Comput. Math. Appl., **61** (2011), 2793–2798

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