A constructive approach to coupled fixed point theorems in metric spaces

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ABSTRACT.

In this paper we establish the existence and uniqueness of a coupled fixed point for operators $F : X \times X \to X$ satisfying a new type of contractive condition, which is weaker than all the corresponding ones studied in literature so far. We also provide constructive features to our coupled fixed point results by proving that the unique coupled fixed point of F can be approximated by means of two distinct iterative methods: a Picard type iterative method of the form $x_{n+1} = F(x_n, x_n)$, $n \geq 0$, with $x_0 \in X$, as well as a two step iterative method of the form $y_{n+1} = F(y_{n-1}, y_n)$, $n \geq 0$, with $y_0, y_1 \in X$. We also give appropriate error estimates for both iterative methods. Essentially we point out that all coupled fixed point theorems existing in literature, that establish the existence and uniqueness of a coupled fixed point with equal components, could be derived in a much more simpler manner.

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