CARPATHIAN J. MATH. **31** (2015), No. 3, 307 - 312
Online version available at http://carpathian.ubm.ro
Print Edition: ISSN 1584 - 2851 Online Edition: ISSN 1843 - 4401

Common fixed points for an uncountable family of weakly contractive operators

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ABSTRACT. In this paper, we consider some behavior concerning common fixed points of an uncountable family of operators. We apply here the concept of circular metric spaces, and the operators are assumed to satisfy different rates of weak contractivity. We show under certain assumptions that weakly contractive family have some strong relationships to its common selector in terms of their fixed points.

1. INTRODUCTION

Suppose that $\{F_t\}_t$ is a family of (set-valued) operators. The common fixed point problem of $\{F_t\}_t$ is to determine whether the intersectional property $x \in \bigcap_t F_t(x)$ holds for at least one x.

It was studied by Frigon [8] the contractive family $\mathcal{F}$ on a gauge space (for basic definitions and properties, consult [4]) such that: if some $f_{t_0} \in \mathcal{F}$ has a fixed point, then every $f_t \in \mathcal{F}$ has a fixed point. This result is then improved by Espínola and Kirk [5], ensuring the common fixed point of a contractive family. For more results in this direction, see *e.g.* [7, 6] and references therein.

Very recently, we introduced the concept of a circular metric space [2, 3] and consider some nonlinear problems that involve the non-nullity of an uncountable intersection of certain sets, including the common fixed point problems for a contractive family.

Similar to gauge spaces, a circular metric space is an optional choice that allows one to assign different measurements to different objects so that only delicate bonds were formed between them. This benefit has made the concept a lot more accessible to users in both practical and theoretical ways.

In this paper, we consider an uncountable (continuum, to be precise) family $\{F_t\}_{t>0}$ of operators, and then invoke the weak contractivity with respect to the circular metrics $\mathcal{W}$ and the modifiers of class Φ, whereby the operators are associated to different members in $\mathcal{W}$ and modifiers in Φ. Our main theorem guarantees under particular assumptions that $\{F_t\}_{t>0}$ has a unique common selector f, and that $\mathrm{Fix}(f) = \bigcap_{t>0} \mathrm{Fix}(F_t)$. Moreover, we have the coexisting behavior between common fixed points of $\{F_t\}_{t>0}$ and some special point x_0 whose orbit is proved to be convergent to one of the common fixed points of $\{F_t\}_{t>0}$.

2. CIRCULAR METRIC SPACES

This section is objected to recollect the basic notions and properties of circular metric spaces. Note that some notations will be varied and simplified for the convenience of this paper. No difference were made in terms of general meanings and usages.

Received: 30.08.2014; In revised form: 02.03.2015; Accepted: 12.03.2015
2010 *Mathematics Subject Classification.* 47H10, 47H09.
Key words and phrases. *Common fixed point, weakly contractive operators, circular metric spaces.*
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Definition 2.1 ([3, 2]). Let X be a nonempty set. A family $\mathcal{W}$ of functions $w_t : X \times X \to [0, \infty]$, indexed on $t > 0$, is said to be a *circular metric* on X if the following properties are satisfied:

(C1) $(\forall x, y \in X)\ w_t(x, y) = 0$ for all $t > 0 \iff x = y$.
(C2) $(\forall x, y \in X, \forall t > 0)\ w_t(x, y) = w_t(y, x)$.
(C3) $(\forall x, y, z \in X, \forall t > 0)$ There is $0 < s < t$ such that $w_t(x, y) \leq w_s(x, z) + w_{t-s}(z, y)$.

The pair $(X, \mathcal{W})$ is then called a *circular metric space*.

Typical examples of this space may be found in [2, 3].

Let $(X, \mathcal{W})$ be a circular metric space, $x \in X$, and $r > 0$. We define the open ball around x of radius r by

$$B(x; r) := \left\{ z \in X,\ \sup_{t>0} w_t(x, z) < r \right\}.$$

We shall always make use of the Hausdorff topology $\tau_{\mathcal{W}}$ which is generated from these balls.

Definition 2.2 ([2, 3]). Let $(X, \mathcal{W})$ be a circular metric space, $E \subset X$, and $t > 0$. We shall use the following terminology and notations:

(i) The *t-diameter* of E is defined by $\operatorname{diam}_t(E) := \sup_{x,y \in E} w_t(x, y)$.
(ii) E is said to be *t-bounded* if $\operatorname{diam}_t(E) < \infty$.
(iii) E is said to be *t-proximinal* if for each $x \in X$, the infimum $\inf_{y \in E} w_t(x, y)$ is attained for some $\bar{y} \in E$.
(iv) The associated Pompeiu-Hausdorff circular metric on $\mathcal{C}(X)$ (see [1] for more details), the family of all nonempty closed subsets of X, is then defined to be the family $\overline{\mathcal{W}} := \{W_t\}_{t>0}$ by

$$W_t(E, F) := \max \left\{ \inf_{e \in E} \sup_{f \in F} w_t(e, f),\ \inf_{f \in F} \sup_{e \in E} w_t(e, f) \right\}, \quad \forall E, F \in \mathcal{C}(X).$$

Proposition 2.1 ([2]). *Suppose that $(X, \mathcal{W})$ is a circular metric space, and $E, F \subset X$ is t-proximinal for every $t > 0$. Then for each $e \in E$, $f \in F$, and $t > 0$, there exists $0 < s < t$ such that*

$$w_t(e, f) \leq W_t(E, F) + \operatorname{diam}_{t-s}(F).$$

Definition 2.3 ([2]). *Let $(X, \mathcal{W})$ be a circular metric space, and $t > 0$. We say that a sequence $(x_n) \subset X$ is t-*Cauchy *if for each $\epsilon > 0$, we have $w_t(x_m, x_n) < \epsilon$ for sufficiently large $m, n \in \mathbb{N}$.*

3. Main theorem

In this section, we shall always assume that $(X, \mathcal{W})$ is a circular metric space with the following properties:

(A1) X is complete in the sense that: if $(x_n) \subset X$ is t-Cauchy for all $t > 0$, then there is a unique point $x \in X$ such that $\lim_{n \longrightarrow \infty} w_t(x_n, x) = 0$ for every $t > 0$.
(A2) $\mathcal{W}$ is a decreasing family of circular metrics, *i.e.* $w_{(\cdot)}(x, y)$ is decreasing for fixed $x, y \in X$.
(A3) w_t is l.s.c. in $X \times X$ for t sufficiently small.
(A4) $s = \frac{t}{2}$ holds for each $t > 0$ in (C3).

We also assume that Φ is the class of continuous functions $\phi : [0, \infty) \to [0, \infty)$ satisfying:

(Φ 1) $\phi^{-1}(0) = \{0\}$ and $\phi(t) < t$ for $t > 0$.

(Φ 2) both ϕ and $\mathrm{id} - \phi$ are increasing with $\lim_{t\longrightarrow\infty}\phi(t) = \infty$, where id denotes the identity function on $[0,\infty)$.

Theorem 3.1. *For each $t > 0$, let $\phi_t \in \Phi$, and let $F_t : X \multimap X$ be an operator whose values are nonempty, closed, s-bounded and s-proximinal for all $s > 0$. Assume that the following conditions are satisfied:*

(i) *For each $t > 0$,*

$$W_t(F_t(x), F_t(y)) \leq w_{4t}(x,y) - \phi_t(w_{4t}(x,y)),$$

whenever $x, y \in X$ are points whose qualities $W_t(F_t(x), F_t(y))$ and $w_{4t}(x,y)$ are finite.

(ii) *If $0 < s \leq t$, then F_t is an enlargement of F_s.*

(iii) *Given $t, \epsilon > 0$, there is $0 < s < t$ such that*

$$\mathrm{diam}_s(F_s(x)) \leq \tfrac{1}{2}\phi_s(\epsilon), \quad \forall x \in X.$$

(iv) *there exists $x_0 \in X$ such that $w_t(x_0, y) < \infty$ for every $y \in F_t(x_0)$ and every $t > 0$.*

Then, we have the following:

(1) *$\{F_t\}_{t>0}$ has a unique common selection, say f.*

(2) $\mathrm{Fix}(f) = \bigcap_{t>0}\mathrm{Fix}(F_t) \neq \emptyset$.

Proof. For $x \in X$, we may see that $\{F_t(x)\}_{t>0}$ is a family of closed and bounded sets. Assumptions (ii), (iii), and the fact that $\phi_s(t) \longrightarrow 0$ (as $t \longrightarrow 0$, with $s > 0$ fixed) guarantee that $\bigcap_{t>0} F_t(x)$ is nonempty and is singleton. Therefore, the operator $f : X \to X$ such that $\bigcap_{t>0} F_t(x) = \{f(x)\}$ defines the unique common selection of the family $\{F_t\}_{t>0}$.

Let x_0 be a point regarding (iv), and let (x_n) be the orbit of f around x_0, *i.e.* $x_n = f^n(x_0)$ for $n \in \mathbb{N}$. Assume without loss of generality that $x_m \neq x_n$ for all $m, n \in \mathbb{N}$. In this case, we write

$$\begin{cases} O(x_0; n) := \{x_0, x_1, \ldots, x_n\}, \\ O(x_0) := \{x_0, x_1, \ldots\}. \end{cases}$$

We shall now show that (x_n) is t-bounded for all $t > 0$, that is, we show that

$$\mathrm{diam}_t(O(x_0)) < \infty, \quad \forall t > 0.$$

Let $t > 0$. Set $\epsilon := w_{4t}(x_0, x_1)$, we may find from (iii) the parameter $0 < s < 2t$ such that

$$\mathrm{diam}_{s/2}(F_{s/2}(x)) \leq \tfrac{1}{2}\phi_{s/2}(\epsilon), \quad \forall x \in X.$$

Let $n \in \mathbb{N}$ and $i, j \in \{1, 2, \ldots, n\}$. Observe that

$$\begin{aligned} w_{2s}(x_i, x_j) &\leq w_s(x_i, x_j) \\ &\leq W_{s/2}(F_{s/2}(x_{i-1}), F_{s/2}(x_{j-1})) + \mathrm{diam}_{s/2}(F_{s/2}(x_{j-1})) \\ &\leq w_{2s}(x_{i-1}, x_{j-1}) - \phi_{s/2}(w_{2s}(x_{i-1}, x_{j-1})) + \tfrac{1}{2}\phi_{s/2}(\epsilon) \\ &= (\mathrm{id} - \phi_{s/2})(w_{2s}(x_{i-1}, x_{j-1})) + \tfrac{1}{2}\phi_{s/2}(\epsilon) \\ &\leq (\mathrm{id} - \phi_{s/2})(\mathrm{diam}_{2s}(O(x_0; n))) + \tfrac{1}{2}\phi_{s/2}(\mathrm{diam}_{4t}(O(x_0; n))) \\ &\leq (\mathrm{id} - \tfrac{1}{2}\phi_{s/2})(\mathrm{diam}_{2s}(O(x_0; n))) \\ &< \mathrm{diam}_{2s}(O(x_0; n)) \\ &\leq \mathrm{diam}_s(O(x_0; n)). \end{aligned}$$

From the above arrays, we have the following three important inequalities for each $i, j \in \{1, 2, \dots, n\}$:

$$\begin{cases} w_s(x_i, x_j) < \operatorname{diam}_s(O(x_0; n)), \\ w_{2s}(x_i, x_j) < \operatorname{diam}_{2s}(O(x_0; n)), \\ w_s(x_i, x_j) \leq (\operatorname{id} - \frac{1}{2}\phi_{s/2})(\operatorname{diam}_{2s}(O(x_0; n))). \end{cases}$$

Subsequently, for some $k \in \{1, 2, \dots, n\}$, we have

$$\begin{aligned} \operatorname{diam}_{2s}(O(x_0; n)) &= w_{2s}(x_0, x_k) \\ &\leq w_s(x_0, x_1) + w_s(x_0, x_1) \\ &\leq w_s(x_0, x_1) + (\operatorname{id} - \tfrac{1}{2}\phi_{s/2})(\operatorname{diam}_{2s}(O(x_0; n))). \end{aligned}$$

So, we obtain

$$\phi_{s/2}(\operatorname{diam}_{2s}(O(x_0; n))) \leq w_s(x_0, x_1).$$

Moreover, we get

$$\begin{aligned} \lim_{n \longrightarrow \infty} \phi_{s/2}(\operatorname{diam}_{4t}(O(x_0; n))) &\leq \lim_{n \longrightarrow \infty} \phi_{s/2}(\operatorname{diam}_{2s}(O(x_0; n))) \\ &= \lim_{n \longrightarrow \infty} \phi_{s/2}(\operatorname{diam}_{2s}(O(x_0; n))) \\ &\leq w_s(x_0, x_1) \\ &< \infty. \end{aligned}$$

By the property of the class Φ, the (real) sequence $(\operatorname{diam}_{4t}(O(x_0; n)))$ is bounded above. Hence, it converges to $\operatorname{diam}_{4t}(O(x_0)) < \infty$. This shows that (x_n) has t-bounded orbit for all $t > 0$.

Next, we show that (x_n) is t-Cauchy for all $t > 0$. Let us assume to the contrary that (x_n) is not r-Cauchy at some $r > 0$. For $n \in \mathbb{N}$, set $Q_n := \{x_n, x_{n+1}, \dots\}$. We may see that $(\operatorname{diam}_t(Q_n))$ decreases to some $\delta_t \geq 0$. However, as (x_n) is not r-Cauchy, we have $\delta_r > 0$. Choose accordingly to (iii) the parameter $0 < \ell < r$ such that

$$\operatorname{diam}_{\ell/2}(F_{\ell/2}(x)) \leq \tfrac{1}{2}\phi(\delta_r), \quad \forall x \in X.$$

Suppose that $n \in \mathbb{N}$ and $x_p, x_q \in Q_n$. We may see that

$$\begin{aligned} w_\ell(f(x_p, x_q)) &\leq W_{\ell/2}(F_{\ell/2}(x_p), F_{\ell/2}(x_q)) + \operatorname{diam}_{\ell/2}(F_{\ell/2}(x_q)) \\ &\leq (\operatorname{id} - \tfrac{1}{2}\phi_{\ell/2})(\operatorname{diam}_\ell(Q_n)). \end{aligned}$$

We thus have

$$\operatorname{diam}_\ell(Q_{n+1}) \leq (\operatorname{id} - \tfrac{1}{2}\phi_{\ell/2})(\operatorname{diam}_\ell(Q_n)).$$

Taking $n \longrightarrow \infty$ and using the property of the class Φ, we get $\delta_\ell = 0$. However, we also have $0 = \delta_\ell \geq \delta_r > 0$, which is a contradiction. Therefore, (x_n) is t-Cauchy for every $t > 0$. By the completeness of X, there is a unique $\bar{x} \in X$ such that $\lim_{n \longrightarrow \infty} w_t(x_n, \bar{x}) = 0$ for each $t > 0$.

We next prove that $\bar{x}$ is a fixed point of f. Let us assume that $\bar{x} \neq f(\bar{x})$, so that $w_\nu(\bar{x}, f(\bar{x})) > 0$ for some $\nu > 0$. As $\mathcal{W}$ is a decreasing family, we may also assume that ν is small enough so that w_ν is l.s.c. Note that

$$\begin{aligned} w_\nu(\bar{x}, f(\bar{x})) &\leq w_{\nu/2}(\bar{x}, f(x_n)) + w_{\nu/2}(f(x_n), f(\bar{x})) \\ &\leq w_{\nu/2}(\bar{x}, f(x_n)) + w_{\nu/4}(F_{\nu/4}(x_n), F_{\nu/4}(\bar{x})) + \operatorname{diam}_{\nu/4}(F_{\nu/4}(\bar{x})). \end{aligned}$$

Passing $n \longrightarrow \infty$, we get

$$w_\nu(\bar{x}, f(\bar{x})) \leq \operatorname{diam}_{\nu/4}(F_{\nu/4}(\bar{x})) < \infty.$$

Now, as of (iii), we can choose $0 < \mu < \nu$ satisfying the following inequality:

$$\operatorname{diam}_{\mu/2}(F_{\mu/2}(x)) \leq \tfrac{1}{2}\phi(w_\nu(\bar{x}, f(\bar{x}))), \quad \forall x \in X.$$

With this, we may obtain

$$\begin{aligned} w_\nu(f(x_n), f(\bar{x})) &\leq w_\mu(f(x_n), f(\bar{x})) \\ &\leq W_{\mu/2}(F_{\mu/2}(x_n), F_{\mu/2}(\bar{x})) + \operatorname{diam}_{\mu/2}(F_{\mu/2}(\bar{x})) \\ &\leq w_{2\mu}(x_n, \bar{x}) - \phi_{\mu/2}(w_{2\mu}(x_n, \bar{x})) + \tfrac{1}{2}\phi(w_\nu(\bar{x}, f(\bar{x}))). \end{aligned}$$

Letting $n \longrightarrow \infty$ and using the properties of the class Φ and the semicontinuity of w_ν, we have

$$w_\nu(f(x_n), f(\bar{x})) < w_\nu(f(x_n), f(\bar{x})),$$

which is a contradiction. Therefore, we conclude that $\bar{x} \in \operatorname{Fix}(f)$. The truth that $\operatorname{Fix}(f) = \bigcap_{t>0} \operatorname{Fix}(F_t)$ follows immediately, as $\bigcap_{t>0} F_t(x)$ is always singleton. □

We shall explicitly give a particular example to help illustrating our main result. Some lengthy routine calculations are however excluded, as it may ended up misled.

Example 3.1. Let $X := [0, 1]$. We define for each $k \in \mathbb{N} \cup \{0\}$ and $t > 0$ the following:

$$\begin{cases} I_0 := \left[0, \frac{1}{2}\right] \cup \{1\}, \\ I_k := \left(1 - \frac{1}{2^k}, 1 - \frac{1}{2^{k+1}}\right], \end{cases} \qquad \begin{cases} J_0^t := \{0\} \cup \left[\frac{1}{2(1+4t)}, \frac{1}{1+4t}\right], \\ J_k^t := \left[\frac{1}{2^{k+1}(1+4t)}, \frac{1}{2^k(1+4t)}\right). \end{cases}$$

Note that $X = \bigcup_{k=0}^{\infty} I_k$, and $\left[0, \frac{1}{1+4t}\right] = \bigcup_{k=0}^{\infty} J_k^t$ for every $t > 0$. Moreover, for each $x \in X$, we write $\kappa(x)$ to denote the unique index such that $x \in I_{\kappa(x)}$.

On X, we may define the circular metric $\mathcal{W} := \{w_t\}_{t>0}$ in the following (with $t > 0$):

$$w_t(x, y) := \begin{cases} \frac{1}{1+t}|x - y|, & \kappa(x) = \kappa(y), \\ +\infty, & \text{otherwise.} \end{cases}$$

Clearly, the space $(X, \mathcal{W})$ satisfies the conditions (A1) - (A4).

Again, for each $t > 0$, let ϕ_t be a homeomorphism from $\left[0, \frac{1}{1+4t}\right]$ onto $\left[0, \frac{1}{2(1+4t)}\right]$ such that $\phi_t(0) := 0$, and ϕ_t maps the interval $\left[\frac{1}{2^{k+1}(1+4t)}, \frac{1}{2^k(1+4t)}\right]$ onto another interval $\left[\sum_{i=k+1}^{\infty} \frac{1}{2^{2i+2}(1+4t)}, \sum_{i=k}^{\infty} \frac{1}{2^{2i+2}(1+4t)}\right]$ naturally (*i.e.*, by using only translation and scaling), for all $k \in \mathbb{N} \cup \{0\}$. The graph $\operatorname{Gr}(\phi_t)$ is simply the plane polygonal segment joining the points $\left(\frac{1}{2^k(1+4t)}, \sum_{i=k}^{\infty} \frac{1}{2^{2i+2}(1+4t)}\right)$, $k \in \mathbb{N} \cup \{0\}$, plus the origin $(0, 0)$. It is worth mentioning that on each interval $\left[\frac{1}{2^{k+1}(1+4t)}, \frac{1}{2^k(1+4t)}\right]$, the function ϕ_t restricted to this interval is a linear function whose gradient is $\frac{1}{2^{k+1}}$. Notice that both ϕ_t and $\mathrm{id} - \phi_t$ are increasing, $\phi_t^{-1}(0) = \{0\}$, and $\phi_t(s) < s$ for all $s \in \left[0, \frac{1}{1+4t}\right]$. It is obvious that ϕ_t's can be extended unto $[0, \infty)$, and that the extensions are of the class Φ. We shall remain writing ϕ_t's for such extensions, without causing any ambiguity.

For a given $t > 0$, let $F_t : X \multimap X$ be a map defined by

$$F_t(x) := \left[0, \frac{1}{2^{\max\{1,1/t\}}} \cdot \left(1 - \left(\frac{1+t}{1+4t}\right)\left(1 - \frac{1}{2^{\kappa(x)+2}}\right)x\right)\right], \quad x \in X.$$

Then, F_t's are closed, s-bounded, and s-proximinal for every $s > 0$. Moreover, it is more or less trivial to see that the assumptions (ii) -(iv) are satisfied. Suppose that $t > 0$ is given, and $x, y \in X$ are arbitrary points with $W_t(F_t(x), F_t(y)) < \infty$ and $w_t(x,y) < \infty$. Without losing generality, assume that $x > y$. Observe that

$$\begin{aligned} & W_t(F_t(x), F_t(y)) \\ = \ & \frac{1}{2^{\max\{1,1/t\}}(1+t)} \left| \left(1 - \left(\frac{1+t}{1+4t}\right)\left(1 - \frac{1}{2^{\kappa(x)+2}}\right)x\right) - \left(1 - \left(\frac{1+t}{1+4t}\right)\left(1 - \frac{1}{2^{\kappa(x)+2}}\right)y\right) \right| \\ = \ & \frac{1}{2^{\max\{1,1/t\}}(1+4t)} \left[(x-y) - \frac{1}{2^{\kappa(x)+2}}(x-y) \right] \\ \leq \ & w_{4t}(x,y) - \phi_t(w_{4t}(x,y)). \end{aligned}$$

The last inequality came mainly from the fact that if $x, y \in I_k$, then $\frac{1}{1+4t}(x-y) \in J_k^t$. Therefore, the assumption (i) holds. It thus follows that every prerequisites of our theorem are satisfied.

We may see that the only common selection for $\{F_t\}_{t>0}$ is the zero map $f(x) := 0$, $x \in X$. Moreover, we have $\mathrm{Fix}(f) = \bigcap_{t>0} \mathrm{Fix}(F_t) = \{0\}$, complying with our theorem.

4. Conclusion

In a circular metric space, we deduce the conditions under which a unique common selector of a weakly contractive family is guaranteed. We show some connection between the fixed point set of the selector and of the weakly contractive family. Our main theorem extends the results in [2].

Acknowledgements. Mr. Parin Chaipunya was supported jointly by the Thailand Research Fund (TRF) and King Mongkuts University of Technology Thonburi (KMUTT) through the Royal Golden Jubilee Ph.D. Program (Grant No. PHD/0045/2555).

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