

# On the theory of fixed point theorems for convex contraction mappings

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## ABSTRACT.

Based on the concepts and problems introduced in [Rus, I. A., *The theory of a metrical fixed point theorem: theoretical and applicative relevances*, Fixed Point Theory, **9** (2008), No. 2, 541–559], in the present paper we consider the theory of some fixed point theorems for convex contraction mappings. We give some results on the following aspects: data dependence of fixed points; sequences of operators and fixed points; well-posedness of a fixed point problem; limit shadowing property and Ulam-Hyers stability for fixed point equations.

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