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A new type of contractions that characterize metric completeness

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ABSTRACT.

We prove that a new type of contractions characterizes the metric completeness of the underlying space. We also discuss the Meir-Keeler fixed point theorem.

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