

A new approach to α - ψ -contractive mappings and generalized Ulam-Hyers stability, well-posedness and limit shadowing results

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ABSTRACT.

In this paper, we introduce the new concept of weakly α -admissible mapping and give example to show that our concept is different from the concept corresponding existing in the literature. We also establish fixed point theorems by using such concept along with α - ψ -contractive condition and give some example which support our main result while previous results in literature are not applicable. Moreover, we study the generalized Ulam-Hyers stability, the well-posedness and the limit shadowing for fixed point problems satisfy our conditions.

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