

Common fixed points of multivalued F -contractions on metric spaces with a directed graph

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ABSTRACT.

In this paper, we establish the existence of common fixed points of multivalued F -contraction mappings on a metric space endowed with a graph. An example is presented to support the results proved herein. Our results unify, generalize and complement various known comparable results in the literature.

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