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Second order differential equations with an irregular singularity at the origin and a large parameter: convergent and asymptotic expansions

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ABSTRACT.

We consider the second order linear differential equation

$$y'' = \left[\frac{\Lambda^2}{t^\alpha} + g(t)\right] y,$$

where Λ is a large complex parameter and g is a continuous function. In previous works we have considered the case $\alpha \in (-\infty, 2]$ and designed a convergent and asymptotic method for the solution of the corresponding initial value problem with data at $t = 0$. In this paper we complete the research initiated in those works and analyze the remaining case $\alpha \in (2, \infty)$. We use here the same fixed point technique; the main difference is that for $\alpha \in (2, \infty)$ the convergence of the method requires that the initial datum is given at a point different from the origin; for convenience we choose the point at the infinity. We obtain a sequence of functions that converges to the unique solution of the problem. This sequence has also the property of being an asymptotic expansion for large Λ (not of Poincaré-type) of the solution of the problem. The generalization to non-linear problems is straightforward. An application to a quantum mechanical problem is given as an illustration.

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