

# A note on some positive linear operators associated with the Hermite polynomials

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## ABSTRACT.

In this paper we give direct approximation theorems and the Voronovskaya type asymptotic formula for certain linear operators associated with the Hermite polynomials. These operators extend the well-known Szász-Mirakjan operators.

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## REFERENCES

- [1] Andrews, L. C., *Special Functions for Engineers and Applied Mathematicians*, Mac Millan, New York, 1985
- [2] Appell, P. and Kampé de Fériet, J., *Fonctions Hypergéométriques et Hypersphériques; Polynômes d'Hermite*, Gauthier-Villars, Paris 1926
- [3] Babusci, D., Dattoli, G. and Quattromini, M., *On integrals involving Hermite polynomials*, Appl. Math. Lett., **25** (2012), 1157–1160
- [4] Bărbosu, D., *Generalized blending operators of Favard-Szasz type*, Bul. Ştiinţ. Univ. Baia Mare Ser. B, **10** (1994), No. 1-2, 45–51
- [5] Bărbosu, D., *Some applications of Shisha-Mond theorem*, Creat. Math. Infor., **23** (2014), No. 2, 141–146
- [6] Bărbosu, D., Pop, O. T. and Miclăuş, D., *On some extensions for the Szász-Mirakjan operators*, An. Univ. Oradea Fasc. Mat., **18** (2011), No. 1, 179–187
- [7] Bărbosu, D., Pop, O. T. and Miclăuş, D., *The Kantorovich form of some extensions for Szász-Mirakjan operators*, Rev. Anal. Numér. Théor. Approx., **39** (2010), No. 1, 8–20
- [8] Bardaro, C. and Mantellini, I., *Approximation properties in abstract modular spaces for a class of general sampling operators*, Applicable Analysis, **85** (2006), No. 4, 383–413
- [9] Bardaro, C. and Mantellini, I., *A quantitative asymptotic formula for a general class of discrete operators*, Comput. Math. Appl., **60** (2010), No. 10, 2859–2870
- [10] Bardaro, C. and Mantellini, I., *Asymptotic formulae for bivariate discrete operators: applications to generalized sampling series and Szász-Mirakjan operators*, Panamer. Math. J., **20** (2010), No. 1, 1–21
- [11] Bardaro, C. and Mantellini, I., *A Voronovskaya type theorem for a general class of discrete operators*, Rocky Mountain J. Math., **39** (2009), No. 5, 1411–1442
- [12] Becker, M., *Global approximation theorems for Szász-Mirakjan and Baskakov operators in polynomial weight spaces*, Indiana Univ. Math. J., **27** (1978), No. 1, 127–142
- [13] Becker, M., Kucharski, D. and Nessel, R. J., *Global approximation theorems for the Szász-Mirakjan operators in exponential weight spaces*, Linear Spaces and Approximation Proc. Conf. (Oberwolfach, 1977), Birkhäuser Verlag, Basel, Internat. Series of Num. Math., **40** (1978), 319–333
- [14] Butzer, P. L. and Karsli, H., *Voronovskaya-type theorems for derivatives of the Bernstein-Chlodovsky polynomials and Szász-Mirakjan operators*, Comment. Math. **49** (2009), No. 1, 33–58
- [15] Dattoli, G., Srivastava, H. M. and Zhukovsky, K., *Orthogonality properties of the Hermite and related polynomials*, J. Comput. Appl. Math., **182** (2005), 165–172

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- [16] Dattoli, G., *Generalized polynomials, operational identities and their applications*, J. Comput. Appl. Math., **118** (2000), 111–123
- [17] DeVore, R. A. and Lorentz, G. G., *Constructive Approximation*, Springer–Verlag, Berlin, 1993
- [18] Ditzian, Z. and Totik, V., *Moduli of Smoothness*, Springer–Verlag, New York, 1987
- [19] Duman, O. and Özsarlan, M. A., *Szász–Mirakjan type operators providing a better error estimation*, Appl. Math. Lett., **20** (2007), 1184–1188
- [20] Gonska, H., Pițul, P. and Raşa, I., *General King type operators*, Result. Math., **53** (2009), No. 3–4, 279–286
- [21] Johnen, H. and Scherer, K., *On equivalence of  $K$  functional and moduli of continuity and some applications*, Constructive Theory of Functions of Several Variables, Proc. Conf. (Oberwolfach, 1976), Lecture Notes in Mathematics, **571**, Springer-Verlag, 119–140
- [22] Lebedev, N. N., *Special Functions and their Applications*, Dover, New York, 1972
- [23] Louisell, W. H., *Quantum Statistical Properties of Radiation*, Wiley, New York, 1990
- [24] Mahmudov, N. I.,  *$q$ -Szász–Mirakjan operators which preserve  $x^2$* , J. Comput. Appl. Math., **235** (2011), 4621–4628
- [25] Mortici, C., *An extension of the Szász–Mirakjan operators*, An. St. Univ. "Ovidius" Constanta, **17** (2009), No. 1, 137–144
- [26] Păltănea, R., *Optimal estimates with moduli of continuity*, Result. Math., **32** (1997), 318–331
- [27] Pop, O. T., Bărbosu, D. and Miclăuș, D., *The Voronovskaja type theorem for an extension of Szász–Mirakjan operators*, Demonstratio Math., **45** (2012), No. 1, 107–115
- [28] Pop, O. T., Miclăuș, D. and Bărbosu, D., *The Voronovskaja type theorem for a general class of Szász–Mirakjan operators*, Miskolc Math. Notes, **14** (2013), No. 1, 219–231
- [29] Rempulska, L. and Graczyk, S., *Approximation by modified Szász–Mirakjan operators*, JIPAM, Journal Inequal. Pure Appl. Math., **10** (2009), No. 3, art. 61
- [30] Rempulska, L. and Graczyk, S., *On certain class of Szász–Mirakjan operators in exponential weight spaces*, Int. J. Pure Appl. Math., **60** (2010), No. 3, 259–267
- [31] Rempulska, L. and Walczak, Z., *Modified Szász–Mirakjan operators*, Mathematica Balkanica, **18** (2004), 53–63
- [32] Shisha, O. and Mond, B., *The degree of convergence of linear positive operators*, Proc. Nat. Acad. Sci. U.S.A., **60** (1968), 1196–1200
- [33] Szász, O., *Generalizations of S. Bernstein's polynomials to the infinite interval*, J. Res. Nat. Bur. Standars, Sect. B, **45** (1950), 239–245
- [34] Toczek, G. and Wachnicki, E., *On the rate of convergence and the Voronovskaja theorem for the Poisson integrals for Hermite and Laguerre expansions*, J. Approx. Theory, **116** (2002), 113–125
- [35] Totik, V., *Uniform approximation by Szász–Mirakjan type operators*, Acta Math. Hungar., **41** (1983), No. 3–4, 291–307
- [36] Xie, L. and Xie, T., *Approximation theorems for localized Szász–Mirakjan operators*, J. Approx. Theory, **152** (2008), 125–134

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