

A characterization of cone-convex vector-valued functions

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ABSTRACT.

An interesting result in convex analysis, established by J.-P. Crouzeix in 1977, states that a real-valued function defined on a linear space is convex if and only if each function obtained from it by adding a linear functional is quasiconvex. The aim of this paper is to extend this result for vector-valued functions taking values in a partially ordered linear space.

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