

On strongly convex functions

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ABSTRACT.

The main results of this paper give a connection between strong Jensen convexity and strong convexity type inequalities. We are also looking for the optimal Takagi type function of strong convexity. Finally a connection will be proved between the Jensen error term and an useful error function.

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